The Journal of Neuroscience

Neural Integration of Stimulus History Underlies Prediction for Naturalistically Evolving Sequences

Brian Maniscalco, Jennifer L. Lee, Patrice Abry, Amy Lin, Tom Holroyd, and Biyu J. He

Supplementary Note. β = 1.01 sequence aliasing effect.

Because of its power-law shaped power spectral density, fGn sequences never satisfy the Shannon-Whittaker condition [finite bandwidth spectrum, cf. e.g. [1]] and hence, in principle, cannot be sampled. In consequence, the use of discrete time samples, systematically used in practice, implies an aliasing bias. While this aliasing bias can be neglected when H → 1 (or β → 1), it significantly impacts analysis in the other limit, H → 0 (or β → –1) and notably impairs an accurate estimation of H (or β). This is easily understandable as continuous time fGn power spectral density reads: $\Gamma(f) \approx C f^{-\beta}$, with β = 2H – 1, therefore, the smaller H, the larger energy at high frequencies, the larger the impact of aliasing.

While fBm is technically a non-stationary process, its having stationary increments, fGn, implies that the same argument holds for fBm, to which a *stationarized* spectrum $\Gamma(f) \approx C f^{-\beta}$ with β = 2H + 1 can be associated[2, 3]: Thus, aliasing will occur when H → 0 (or β → 1) and impair the estimation of H (or β).

For sequences with intended β = 1.01 (i.e. fBm sequences with H → 0, or β → 1), given the short sample size used here, we found that the value for β recovered from a linear fit of log(power) vs. log(frequency) was around 0.2. This aliasing issue in the generation of β = 1.01 sequences went previously undetected because our method for validating the empirically recovered Hurst exponent following sequence synthesis by using wavelet analysis was indeed able to recover the intended value of H (= 0.005) for each sequence within a margin of error of 0.01[2, 4]. Nonetheless, inspection of the subjects' behavioral discrimination of sequence β (Fig. 2A) suggests that subjects' judgment of trend strength may have been affected by the aliasing effect, as β = 1.01 sequences were judged to have slightly lower trend strength than β = 0.5 sequences on average.

This aliasing effect is potentially an issue for analyses involving tone pitch prediction, since the mathematical computation of expected value depends on the value of β. When the value of β = 1.01 is used, veridical expected values are produced despite the aliasing effect (these are used in our analyses). However, if β = 0.2 were to be used instead, incorrect expected values would result. To control for the possibility that the aliasing effect hindered our analyses, we repeated the analyses demonstrating behavioral (Fig. 2B) and neural (Fig. 5B) sensitivity to p^*_{34} while omitting all β = 1.01 sequences.

The behavioral results were essentially unchanged when omitting β = 1.01 sequences (Fig. S1A), implying that these results are robust against the aliasing effect. The neural results exhibit similar topology and effect sizes in the time windows of 200 – 250 ms and 250 – 300 ms where significant prediction effects were found (Fig. S1B), implying these results were also robust against the aliasing effect. However, the prediction effect in the significant cluster for the 0 – 50 ms time window was markedly reduced when omitting the β = 1.01 sequences. This does not necessarily imply that the originally discovered effect is attributable to the aliasing effect; other considerations, such as reduction in the number of trials, could also have weakened the statistical robustness of the finding. Nonetheless, because the prediction effect in the 0 – 50 ms time window is less robust, we do not emphasize this finding in the main manuscript.

References

1. Papoulis, A. Probability, Random Variables, and Stochastic Processes. (1991).
2. Ciuciu, P., Varoquaux, G., Abry, P., Sadaghiani, S. & Kleinschmidt, A. Scale-Free and Multifractal Time Dynamics of fMRI Signals during Rest and Task. *Frontiers in physiology* **3**, 186 (2012).
3. Eke, A., Herman, P., Kocsis, L. & Kozak, L.R. Fractal characterization of complexity in temporal physiological signals. *Physiol Meas* **23**, R1-38 (2002).
4. Ciuciu, P., Abry, P. & He, B.J. Interplay between functional connectivity and scale-free dynamics in intrinsic fMRI networks. *Neuroimage* **95**, 248-263 (2014).

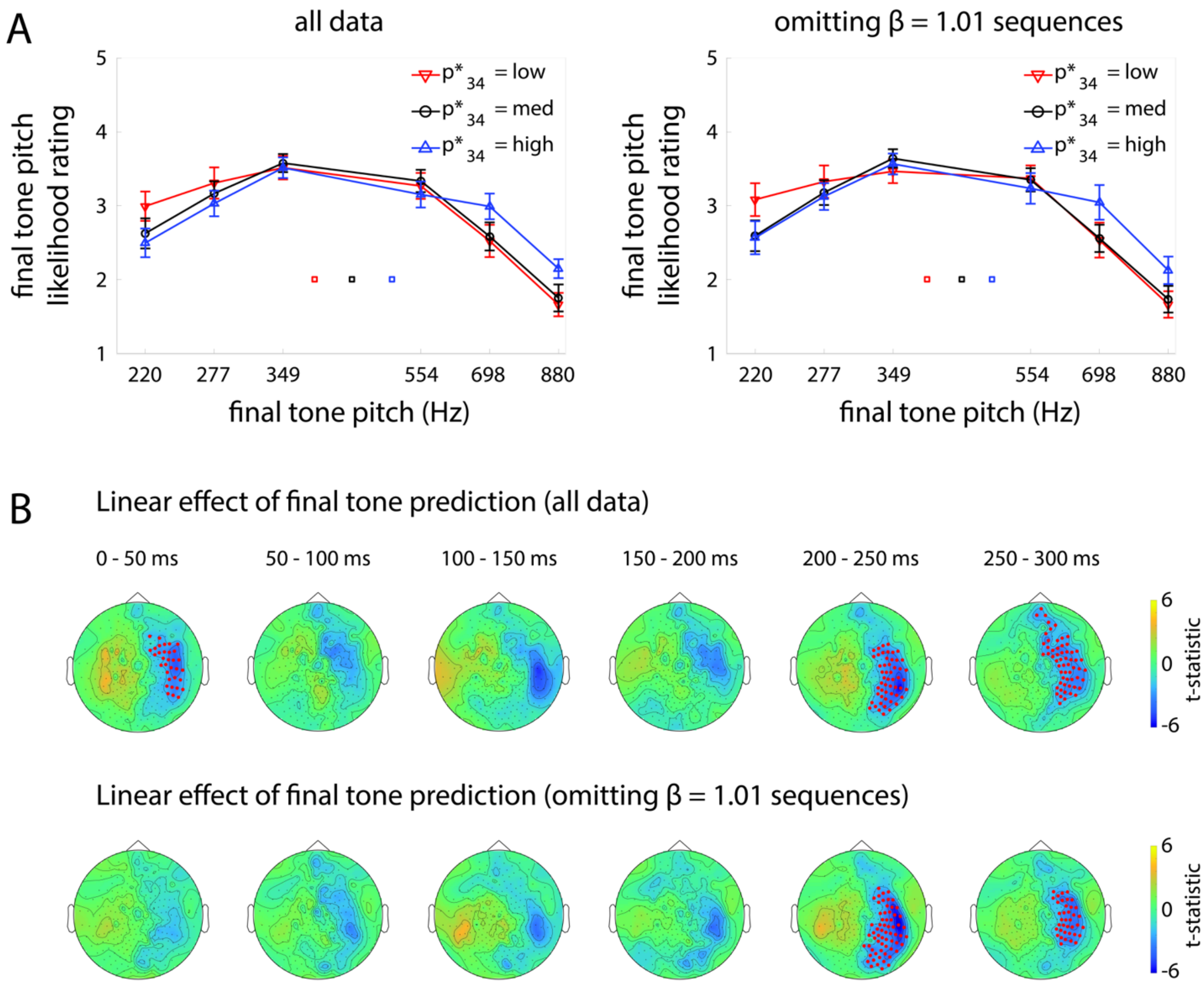

Figure S1. Control analyses for the β = 1 sequence aliasing effect. (A) Final tone pitch likelihood rating. Behavioral results for final tone pitch prediction as reported in Fig. 2B (reproduced here in the left panel) are not affected when omitting β = 1.01 sequences from analysis (right panel). As in the original analysis, a repeated-measures ANOVA on the data with β = 1 trials omitted revealed a significant interaction between p_{34} and p^*_{34} ($F_{10,100} = 4.1$, $p < .001$, $n = 11$). **(B) Neural correlate of final tone pitch prediction.** The neural correlate of final tone pitch prediction found in the (non-baseline-corrected) activity during penultimate tone presentation, as reported in Fig. 5B (reproduced here in the top panel), are very similar when omitting β = 1 sequences from analysis (bottom panel), with the exception of the 0 – 50 ms time window in which a significant cluster found in the original analysis is no longer significant after omitting β = 1 sequences. Red dots indicate significant sensor clusters (all $p < 0.05$, cluster-based permutation test, $n = 11$).