STREAMWISE OSCILLATIONS OF CYLINDERS NEAR THE CRITICAL REYNOLDS NUMBER

I. G. CURRIE AND D. H. TURNBULL

Department of Mechanical Engineering, University of Toronto,
Toronto, Ontario M5S 1A4, Canada

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The purpose of the research reported here is to attempt to clarify the conditions under which in-line oscillations of a circular cylinder occur, and to establish the oscillation characteristics. A mathematical model has been developed which attempts to represent a cylinder vibrating in the streamwise direction. This mathematical model is based on the Van der Pol equation and is similar in many respects to one series of models which exists for cross-flow oscillations. Solutions to the mathematical model indicate the possibility of in-line oscillations and the results so obtained have been fitted to experimental data, thus evaluating the free parameters in the mathematical model.

An experiment was carried out to establish the effects of reduced velocity, Reynolds number, surface roughness, and freestream turbulence on the stability of a circular cylinder to in-line oscillations in the critical Reynolds number range. Results obtained from the test program indicate that, within the range of conditions covered, in-line oscillations occur when the Reynolds number is in the critical range, and then only when free-stream turbulence exists. Surface roughness of the test cylinders did not appear to play a significant role. No attempt was made to investigate in-line oscillations at subcritical Reynolds numbers.

1. INTRODUCTION

Investigations into the nature of flow-induced oscillations of cylinders in the streamwise direction began to appear in the literature after the problems encountered during the construction of an oil jetty at Immingham, England, in the tidal flow of the Humber estuary. The nature of the phenomenon is reported by Sainsbury and King [1], and by Wootton, Warner and Cooper [2]. The main results emerging from the Immingham experience are summarized below:

(a) Flow-induced oscillations in the streamwise direction developed over a Reynolds number range $2 \times 10^5 - 6 \times 10^5$, which is expected to include the critical range.
(b) These oscillations occurred over two distinct ranges of reduced velocity, $V_r = V/f_n D$:
   (i) close to $V_r = 2.0$, accompanied by symmetric vortex shedding;
   (ii) close to $V_r = 2.5$, where alternate vortex shedding was observed.$\dagger$

The response peak at $V_r \approx 2.5$ was easiest to explain, being a simple harmonic of the well documented cross-flow vibration peak in which the frequency of cylinder vibration is twice the vortex shedding frequency. The response peak at $V_r \approx 2.0$ corresponded to a self-

$\dagger$ The nomenclature is given in the Appendix.
excited cylinder oscillation that was completely unexpected. The amplitudes of oscillation ($X_0/D \approx 0.1$) were an order of magnitude smaller than those associated with cross-flow oscillations, but were still large enough to make construction difficult and to potentially damage the jetty structure. Symmetric vortex shedding was observed to accompany the in-line oscillations at $V_*=2.0$, the two symmetric vortices being shed once every cycle of oscillation.

Subsequent investigations have confirmed the existence of two response peaks for in-line oscillations, and have demonstrated that the Reynolds number need not be in the critical range for in-line oscillations to occur. Flow-induced streamwise oscillations of flexible contilevered cylinders in flowing water have been reported by King, Prosser and Johns [3] at Reynolds numbers well below the expected critical range. This leads to the notion that there are two distinct types of in-line oscillation and that they have different fundamental mechanisms.

Based on in-line cylinder oscillations in the critical Reynolds number range, a quasisteady model was proposed, by Martin, Currie and Naudascher [4], to explain a possible mechanism for these oscillations. This model is based on an instability criterion which is only satisfied in the critical Reynolds number range, where the mean drag coefficient versus Reynolds number curve has a negative slope. The model is based on the dynamic equation of motion of the cylinder with a cubic equation representing the experimental variation of drag force versus flow velocity in the critical Reynolds number range.

Almost nothing is known of the mechanism responsible for the symmetric vortex shedding phenomenon associated with the first instability range of in-line oscillations near $V_*=2.0$. In this paper it is proposed that the symmetric vortex shedding is due to a cycle of vortex formation and shedding on a time scale which is short compared with the time scale necessary for the development of a (stable) alternate vortex shedding pattern. If the Reynolds number is low enough when the cylinder is moving downstream, a pair of symmetric vortices (Föppl vortices) form behind the cylinder and remain attached (see Schlichting [5]). As the Reynolds number increases, this pattern of vortex formation breaks down and the vortices are shed alternately. However, if the time scale of vortex formation is long compared with the time scale of the cylinder oscillation, then an alternate pattern may never develop before the Reynolds number again falls. This could account for the observation of a pair of symmetric vortices being shed once each cycle of cylinder oscillation.

In summary, the experimental results reported by Sainsbury and King [1] and by Wootton, Warner and Cooper [2] show that streamwise oscillations occur over a Reynolds number range near the critical range. Furthermore, the stability criterion established by Martin, Currie and Naudascher [4] suggests that one of the mechanisms for streamwise oscillations to exist involves a steep gradient in the drag coefficient versus Reynolds number curve. Such a steep gradient exists in the critical Reynolds number range, and the analysis of Martin, Currie and Naudascher [4] demonstrates that response amplitudes and frequencies may be calculated which are in good agreement with experimental observations. However, the theory which exists requires a detailed knowledge of the shape of the drag coefficient versus Reynolds number curve in the critical range.

One of the objectives of the current research program is to establish a theory which involves more universal parameters than those relating to the shape of the drag coefficient versus Reynolds number curve. Another objective is to provide experimental information relating to the conditions under which streamwise oscillations exist in the critical Reynolds number range.
2. THEORETICAL DEVELOPMENT

2.1. VAN DER POL OSCILLATOR MODEL

Vortex-induced cylinder oscillations in the transverse (or cross-flow) direction have been described mathematically using a lift-oscillator model in which the lift coefficient satisfies the Van der Pol oscillator equation. This mathematical model was introduced by Hartlen and Currie [6], and was subsequently extended by Skop and Griffin [7]. In this paper a similar approach is taken to describe vortex-induced cylinder oscillations in the streamwise (or in-line) direction.

The Van der Pol oscillator model has traditionally been employed in this area of fluid mechanics as a relatively simple description of the periodic separated flow around an oscillating bluff body. The model predicts self-excited (self-limiting) oscillations, the frequency of which can be related to the Strouhal frequency to describe the role of vortex shedding in exciting the cylinder oscillations. The interaction of the cylinder motion with the oscillator is through a forcing term which will be discussed.

2.2. MATHEMATICAL FORMULATION

The physical problem of a cylinder subjected to vortex-induced oscillations in the streamwise direction will be modelled as a damped mass-spring for the equation of the cylinder motion, coupled through the forcing terms to a Van der Pol equation for the instantaneous fluctuating drag coefficient.

The equation of motion in the streamwise direction may be written as

\[ Mx'' + cx' + kx = \text{(forcing)}, \]

where \( M \) is the cylinder mass, including the added mass. The simplest forcing term that could be proposed is \( \frac{1}{2} \rho D L V^2 C_D \), where \( C_D \) is the fluctuating drag coefficient. A somewhat more general forcing term would involve \( C_D' \) as well as \( C_D \), where \( C_D' \) is the mean drag coefficient. The mean drag force term is omitted because it does not contribute to the cylinder oscillations.

Introducing the dimensionless variables \( t = \omega_o t_1 = \sqrt{k/M} t_1 \) and \( x = x_1/D \), this equation can be written in nondimensional form:

\[ \ddot{x} + 2\zeta \dot{x} + x = \text{(forcing)}, \]  

where \( \zeta = c/(2M\omega_o) \) and a dot (\( \cdot \)) denotes differentiation with respect to \( t \).

The instantaneous drag coefficient satisfies a Van der Pol equation:

\[ \ddot{C}_D - \delta \omega_o \dot{C}_D + \frac{\gamma}{\omega_o} \dot{C}_D + \omega_o^2 C_D = \text{(forcing)}, \]

where \( \omega_o \) is the oscillator frequency, and for the second instability range of in-line oscillations is taken to be twice the dimensionless Strouhal frequency \( \omega_n = 2\omega_1/\omega_n = 2f_s/f_n \). The approximate solution to equation (2) for small forcing is

\[ C_D = C_{D0} \sin \omega_o t, \]

where \( C_{D0} = \sqrt{4\delta/3\gamma} \) has a natural physical interpretation as the fluctuating drag coefficient of a stationary cylinder. The value of this parameter is approximately 0.2, as reported by King [8].
The forcing term in equation (1) can be taken to be

$$\alpha_1 C_D + \beta_1 \dot{C}_D,$$

where

$$\alpha_1 = \frac{1}{2} \frac{\rho D L v^2}{M D \omega_n^2} = \frac{\rho D^2 L}{8 \pi^2 M} \left( \frac{V}{f_n} \frac{2 f_x}{1} \frac{1}{2} \right)^2 = \frac{\rho D^2 L}{32 \pi^2 M S_t^2} \omega_n^2 = \alpha \omega_n^2.$$

The constants $\alpha_1$ and $\beta_1$ are assumed to be equal so that the drag variation as well as its absolute value affects the motion of the cylinder. The forcing term in equation (2) is taken to be a linear function of the cylinder displacement and velocity, and is written as

$$ax + b \dot{x},$$

where $a$ and $b$ must be chosen to fit physical data. For harmonic motion the displacement $x$ is proportional to the acceleration, so that the interaction may be thought of as being dependent on both the cylinder velocity and acceleration.

The general model for in-line cylinder oscillations may, therefore, be written as

$$\ddot{x} + 2 \zeta \dot{x} + x = \omega^2 (C_D + \dot{C}_D),$$  \hspace{1cm} (3)

$$\ddot{C}_D - \delta \omega_e \dot{C}_D + \frac{\gamma}{\omega_e} \dot{C}_D + \omega_e^2 C_D = ax + b \dot{x},$$  \hspace{1cm} (4)

where

$$\zeta = \frac{c}{2 M \omega_n},$$  \hspace{1cm} (5)

$$\alpha = \frac{\rho D^2 L}{32 \pi^2 M S_t^2},$$  \hspace{1cm} (6)

$$\sqrt{\frac{4 \delta}{3 \gamma}} \simeq 0.2,$$  \hspace{1cm} (7)

and

$$\omega_n = 2 \frac{\omega_s}{\omega_n} = 2 \frac{f_x}{f_n} - 2 S_t V_r$$  \hspace{1cm} (8)

is assumed to be close to unity.

2.3. MODEL SOLUTIONS

To obtain approximate solutions to equations (3) and (4), it is assumed that all the coefficients are small, of order $O(\varepsilon)$ where $\varepsilon \ll 1$, except $\omega_s$ which is $1 + O(\varepsilon)$. That is, the coefficients are taken to be $\zeta = \varepsilon \zeta_0$, $\alpha = \varepsilon \alpha_0$, etc., and $\omega_s = 1 + \varepsilon \Omega$, where now $\zeta$, $\alpha$, etc, and $\Omega$ are of order $O(1)$. Physically, these assumptions correspond to small damping, coupling (forcing), and small nonlinearities. Under these assumptions, equations of the type (3) and (4) can be solved using a two-time perturbation expansion as described by Cole [9] and by Hall [10]. The introduction of two time scales makes sense physically, as explained previously, if the fast time is equated to the time scale of the cylinder oscillation while the slow time is the time scale of the formation of an alternate vortex pattern.

The assumption is made that $x$ and $C_D$ depend not only on $t$, but also on a slow time $\tau = \varepsilon t$. To order $\varepsilon$, the solution to equations (3) and (4) is assumed to have the form

$$x = x_0(t, \tau) + \varepsilon x_1(t, \tau),$$

$$C_D = C_{D_0}(t, \tau) + \varepsilon C_{D_1}(t, \tau).$$
Substituting this assumed solution into equations (3) and (4), and noticing that

$$\omega_0^2 \approx 1 + 2\alpha \Omega$$

one obtains a hierarchy of equations to be solved:

$$\frac{\partial^2 x_0}{\partial t^2} + x_0 = 0,$$  \hspace{1cm} (9a)

and

$$\frac{\partial^2 x_1}{\partial t^2} + x_1 = -2 \frac{\partial^2 x_0}{\partial t \partial \tau} - 2 \zeta \frac{\partial x_0}{\partial t} + \alpha C_{D_o} + \alpha \frac{\partial C_{D_0}}{\partial t};$$  \hspace{1cm} (9b)

$$\frac{\partial^2 C_{D_0}}{\partial t^2} + C_{D_0} = 0,$$  \hspace{1cm} (10a)

$$\frac{\partial^2 C_{D_1}}{\partial t^2} + C_{D_1} = -2 \frac{\partial^2 C_{D_0}}{\partial t \partial \tau} - \gamma \left( \frac{\partial C_{D_0}}{\partial t} \right)^3 + \delta \frac{\partial C_{D_0}}{\partial t} - 2 \Omega C_{D_0} + \alpha x_0 + b \frac{\partial x_0}{\partial t}.$$  \hspace{1cm} (10b)

The general solutions of equations (9a) and (10a) are

$$x_0(t, \tau) = X_o(\tau) \cos (t + \phi_o(\tau)), \hspace{1cm} (11)$$

$$C_{D_0}(t, \tau) = C_{D_0}(\tau) \cos (t + \psi_o(\tau)).$$  \hspace{1cm} (12)

The standard procedure in the two-time perturbation method is to substitute (11) and (12) into equations (9b) and (10b), and equate the coefficients of \(\cos (t + \phi_o)\), \(\cos (t + \psi_o)\) and \(\sin (t + \phi_o)\), \(\sin (t + \psi_o)\) to zero, in order to avoid “secular” solutions (solutions which diverge as \(t \to \infty\)). When this is done, four equations are obtained for \(X_o(\tau), C_{D_0}(\tau), \phi_o(\tau)\) and \(\psi_o(\tau)\):

\[
\begin{align*}
\frac{dX_o}{d\tau} &= -\zeta X_o + \frac{1}{2} \alpha C_{D_0} (\cos \phi_o - \sin \phi_o),
\frac{d\phi_o}{d\tau} &= \frac{1}{2} \zeta \frac{C_{D_0}}{X_o} (\cos \phi_o + \sin \phi_o),
\frac{dC_{D_0}}{d\tau} &= \frac{1}{2} \delta C_{D_0} - \frac{3}{4} \gamma C_{D_0}^3 + \frac{1}{2} X_o (\cos \phi_o - a \sin \phi_o),
\frac{d\psi_o}{d\tau} &= \Omega - \frac{1}{2} X_o (a \cos \phi_o - b \sin \phi_o),
\end{align*}
\]

where \(\mu_o = \mu_o(\tau) = \phi_o(\tau) - \psi_o(\tau)\).

To obtain steady-state, single-frequency solutions to equations (13), one assumes the response frequency is \(1 + \epsilon \omega_o\), where \(\omega_o\) is of order \(O(1)\). The amplitudes \(X_o\) and \(C_{D_0}\) are assumed constant, while the phases \(\phi_o\) and \(\psi_o\) both drift at the slow rate \(\epsilon \omega_o\). Then equations (13) become

\[
\begin{align*}
\zeta X_o - \frac{1}{2} \alpha C_{D_0} (\cos \mu_o - \sin \mu_o) &= 0, \\
\omega + \frac{1}{2} \zeta \frac{C_{D_0}}{X_o} (\cos \mu_o + \sin \mu_o) &= 0, \\
\frac{1}{2} \delta C_{D_0} - \frac{3}{4} \gamma C_{D_0}^3 + \frac{1}{2} X_o (\cos \mu_o + a \sin \mu_o) &= 0, \\
\omega - \Omega - \frac{1}{2} X_o (a \cos \mu_o - b \sin \mu_o) &= 0.
\end{align*}
\]
Equations (14) can be manipulated to obtain approximate steady-state, single-frequency solutions to equations (13) in the following form:

**Frequency:**

\[ \Omega = \omega + \frac{k_1 \zeta - k_2 \omega}{4(\omega^2 + \zeta^2)}; \quad (15) \]

**Amplitude:**

\[ C_{\theta u}^2 = \frac{4}{3 \gamma} \left( \delta - \frac{k_1 \omega + k_2 \xi}{2(\omega^2 + \zeta^2)} \right), \quad (16) \]

\[ X_u^2 = \frac{\sigma^2}{2(\omega^2 + \zeta^2)} C_{\theta u}^2; \quad (17) \]

**Phase:**

\[ \tan \phi_u = \frac{\omega + \zeta}{\omega - \zeta}. \quad (18) \]

Here \( k_1 = \alpha(a + b) \) and \( k_2 = \alpha(a - b) \). In these solutions, \( \Omega \) and \( \omega \) may be thought of as "detuning" variables: \( \Omega \) represents the detuning between the cylinder natural frequency and the Van der Pol oscillator frequency, while \( \omega \) represents the detuning between the response frequency and the cylinder natural frequency.

The stability of the steady state solutions given by equations (15)–(18) can be checked by linearizing equations (13) about the steady state solution. Positive real parts of eigenvalues of the resulting Jacobian matrix then correspond to unstable solutions of (13).

### 2.4. Model Predictions

The theory outlined in the preceding sections can be utilized, with a suitable choice of model parameters, to predict cylinder oscillations in the streamwise direction. The model parameter \( \alpha \) is determined from equation (6) while the ratio \( \delta/\gamma \) is determined from equation (7). The absolute values of \( \delta \) and \( \gamma \) are adjusted to match experimental data. The damping factor \( \zeta \) can be determined from equation (5) if the damping coefficient \( c \) is known. A more useful formula in the case of small-amplitude vibrations in a high Reynolds number flow is given by Blevins [11]:

\[ \zeta = \frac{1}{4\pi} \left( \frac{\rho D^2 L}{M} \right) \left( \frac{V}{f_c D} \right) C_p, \quad (19) \]

where \( C_p \) is the mean drag coefficient. Finally, since there is no known physical reasoning guiding the choice of model parameters \( a \) and \( b \), these are chosen to fit physical data.

The model can be fitted to data obtained at Immingham, as reported by Wootton, Warner and Cooper [2], in both instability regions of in-line oscillations. The mass ratio of the steel piles used at Immingham can be estimated as

\[ \frac{\rho D^2 L}{M} \approx 0.4. \]

In the second instability region (near \( V_c = 2.5 \)), where the Reynolds number was approximately \( 5 \times 10^5 \), and the mean drag coefficient is estimated to be \( 0.5 \), the model parameters determined from (6), (7) and (19) are \( \zeta = 0.04, \alpha = 0.022, \beta = 0.022, \gamma = 50 \). It is seen that the assumption that all the parameters are of order \( O(1) \) is not strictly adhered to, but reasonable results are obtained, nevertheless. Based on the Van der Pol oscillator model of Hartlen and Currie [6] for cross-flow vibrations, \( a = 0 \) is a sensible choice for the alternate vortex shedding for induced oscillations in the second instability region. The model parameter \( b \) was chosen to fit the Immingham data as reported by Wootton,
Warner and Cooper [2], and was selected to be $b = 4$. Finally, the frequency was translated into reduced velocity using equation (8):

$$V_r = \frac{1 + \varepsilon \Omega}{2St},$$

where the Strouhal number was assumed to be $St = 0.2$. Stable, steady-state solutions were computed using equations (15)–(18), and these are shown in Figure 1 with the Immingham data superimposed.

The Van der Pol oscillator model can also be fitted to the first instability region of vortex-induced, in-line cylinder oscillations (near $V_r = 2.0$). In this case the Reynolds number was approximately $3.5 \times 10^5$, and the mean drag coefficient is estimated to be 0.6. The model parameters determined from (6), (7) and (19) are $\varepsilon = 0.01$, $\zeta = 0.04$, $\alpha = 0.028$, $\delta = 3$, $\gamma = 50$. The main difference in the modelling of the two instability regions of in-line oscillations is the forcing term of the Van der Pol equation. It has been suggested by Wootton, Warner and Cooper [2] that the first instability region might depend on the oscillation amplitude, rather than velocity. This was the motivation for taking $b = 0$ and selecting $a$ to fit the Immingham data. The model parameter $a$ was selected to be $a = 5$. It should be noted that there is no obvious way to translate frequency into reduced velocity in the first instability region. Based on the observation that $V_r \sim 2.0$, the translation to reduced velocity was made rather arbitrarily using

$$V_r = (1 + \varepsilon \Omega)(2).$$
Stable, steady-state solutions, computed using equations (15)–(18), are shown in Figure 2 with the Immingham data superimposed.

The Van der Pol oscillator model shows reasonable agreement with the physical data in the amplitudes of oscillation in both instability regions, and in the reduced velocity range over which the oscillations occur in the second instability region. The fluctuating drag coefficient, $C_{Dv}$, is approximately 0.2, as was desired. It is seen that for the second instability region, the fluctuating drag coefficient takes on a maximum value near the point of maximum oscillation amplitude, while for the first instability region, $C_{Dv}$ is decreasing to a minimum at the point of maximum oscillation amplitude. The model also predicts that the phase difference between the Van der Pol forcing and the cylinder response is close to zero in the first instability region, which is just what one might predict for cylinder oscillations excited by two symmetric vortices being shed once every cycle of oscillation. The corresponding phase difference for the second instability region of in-line oscillations is approximately 45°.

These results indicate that the cylinder oscillations in the second instability region are indeed a simple harmonic of the “velocity driven” transverse cylinder oscillations, while those in the first instability region may be “amplitude driven”.

3. EXPERIMENTAL RESULTS

3.1. Test Facilities

Experiments were performed to study some aspects of streamwise oscillations of circular cylinders. The test rig, operated in a 122 m long towing channel, consisted of an aluminum frame around which two hydrofoil sections were placed, creating a uniform, two-dimensional flow over the test cylinders. This rig was pulled through the water by a towing carriage with very accurate speed control at speeds of up to 6 m/s. The 1.2 m long test cylinders were mounted between the hydrofoils on specially designed bearings which allowed motion of up to 50 mm in amplitude in any direction in a plane perpendicular to the free surface of the water.

Two test cylinders were employed, one being 165 mm in diameter and the other being 356 mm in diameter. The width of the towing tank was 4.68 m and the water depth was 2.74 m. This gave a blackage ratio of 3.24% or less. The models were located 0.91 m below the free surface of the water. Each cylinder could be fitted with a surface roughening layer on which the height of the protuberances was about 1 mm.

For experiments described in this paper, only the horizontal (streamwise) cylinder motions were of interest, so the bearings were clamped in the vertical direction. Support for the test cylinders in the horizontal (streamwise) and vertical (transverse) directions was provided by 1.6 mm cables attached to the bearings, drawn over pulleys of minimal friction, and cleated to the frame of the test rig. Springs of various stiffnesses were connected onto the cables to provide various vibrational frequencies of the cylinders, and load cells were connected at the same point to measure the forces on the cylinder. The motion of the test cylinders was measured with differential transformer displacement transducers which followed the cylinder endplates. The support arrangement for the test cylinder is shown in Figure 3.

The flow over the cylinder could be made turbulent by pulling a screen through the water approximately seven mesh lengths upstream of the cylinder. This screen was designed to provide a “flat” mean velocity profile along the cylinder axis, and a scale of turbulence approximately equal to the cylinder diameter. Measurements of the flow characteristics behind the screen were made with a cylindrical hot-film probe and
constant temperature anemometer. The signals from the hot-film probe, as well as the load cells and displacement transducers, were recorded directly onto an on-board computer, and stored for later analysis.

3.2. TEST RESULTS

Flow-induced, in-line cylinder oscillations were observed for a smooth 356 mm outer-diameter cylinder in turbulent flow. Figure 4 shows a portion of the output from the displacement transducers. The unstable condition occurred when the cylinder natural frequency was $f_n = 0.52$ Hz, the Reynolds number was $Re = 1.7 \times 10^5$ and the reduced velocity was $V_r = 2.6$. This was in the critical Reynolds number range for this cylinder in turbulent flow, and was close to the reduced velocity expected for the second instability region of in-line oscillations.

These single-frequency, in-line cylinder oscillations were observed in the turbulent flow over a reduced velocity range of approximately 2.0–3.0. This also corresponded to the critical Reynolds number range for the cylinder in turbulent flow.

The oscillations observed were very small, indicative of a highly damped system. By
studying the decay of cylinder oscillations in water after release from an initial
displacement, the logarithmic decrement of the system was estimated to be \( \delta_s \approx 1.8 \). The
damping factor can be computed from

\[
\zeta = \frac{\delta_s}{\sqrt{(2\pi)^2 + \delta_s^2}},
\]

and it is found that \( \zeta \approx 0.25 \) (for \( \varepsilon = 0.01 \)). Thus the damping factor is about six times
greater than that which existed at Immingham.

When the turbulence was removed from the flow, but conditions were otherwise
matched to the unstable condition (critical Re, \( V_r \approx 2.5 \)), no in-line cylinder oscillations
were observed. It should be noted that for undisturbed flow the critical Reynolds numbers
defining the critical range are considerably higher than in turbulent flow, and
consequently the springs required to match the reduced velocity are stiffer and provide
more constraint to cylinder motion. With the high damping present in the system, this
added constraint is probably more than enough to prevent oscillations from occurring.

The test cylinder was also surface roughened by glueing to it 1 mm silicon spheres, and
it was tested under the same unstable conditions as the smooth cylinder (turbulent flow,
\( V_r \approx 2.5 \)). In this case no single-frequency, in-line cylinder oscillations were observed. This
lends some support to the explanation, mentioned earlier, that the mechanism responsible
for streamwise cylinder oscillations depends on an instability criterion which is sensitive
to the drag characteristics of the cylinder. Figure 5 shows the drag coefficient as measured
for the test cylinder. The results show that the steepest slope and the minimum absolute
value of the drag are associated with a smooth surface in turbulent flow.

The Van der Pol oscillator model can be fitted to the data obtained in these
experiments. Using equations (6), (7) and the value of \( \zeta \) determined experimentally above,
the model parameters are \( \varepsilon = 0.01, \; \zeta = 0.25, \; \alpha = 1, \; \beta = 1, \; \gamma = 50, \) and \( b \) is taken to be
\( b = 4 \). The parameter \( a \) was chosen to be zero since the observed oscillations occurred
near the reduced velocity range expected for the second instability region if in-line
oscillations. The stable, steady-state model solution is shown in Figure 6 with values of the nondimensionalized amplitude \((X_0/D)\) from the experiments superimposed.

4. CONCLUSIONS

A Van der Pol oscillator model has been adapted to flow-induced cylinder oscillations in the streamwise direction, and approximate model solutions have been computed by taking into account two inherent time scales in the problem. The model has been fitted to experimental data in the two instability regions of in-line oscillations with good agreement in the amplitude of oscillation. The model predictions make sense when viewed against the physical characteristics of the two instability regions.

Experimental data have been obtained which lends support to an explanation for one form of streamwise cylinder oscillations which is based on an instability criterion satisfied only in the critical Reynolds number range. It appears that the occurrence of in-line oscillations may depend on a number of conditions (critical Re, critical \(V_r\), damping threshold, freestream turbulence level) being met simultaneously. The other form of in-line oscillation was not observed, presumably due to the high level of damping.

REFERENCES


APPENDIX: LIST OF SYMBOLS

$\bar{C}_D$ mean drag coefficient (mean drag force $= \frac{1}{2} \rho DLV^2 \bar{C}_D$)
$D$ cylinder diameter
$f_n$ cylinder natural frequency ($\omega_n = 2\pi f_n$)
$f_s$ vortex shedding or Strouhal frequency ($\omega_s = 2\pi f_s$)
$L$ cylinder length
$M$ cylinder mass (including added mass)
$Re$ Reynolds number, $VD/\nu$ ($\nu$ = fluid kinematic viscosity)
$S$ Strouhal number, $f_s D/V$
$V$ mean flow speed
$V_r$ reduced velocity, $V/f_s$
$X_o$ amplitude of cylinder oscillation
$\varepsilon$ parameter defined in equation (6)
$\gamma$ parameter defined in equation (2)
$\delta$ parameter defined in equation (2)
$\zeta$ damping parameter, defined in equation (5)
$\rho$ fluid density
$\omega_n$ cylinder natural frequency ($\omega_n = 2\pi f_n$)
$\omega_s$ vortex shedding or Strouhal frequency ($\omega_s = 2\pi f_s$)
$\omega_0$ dimensionless Strouhal frequency ($\omega_0 = 2\omega_s/\omega_n$)